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Magnetohydrodynamic source flow between two parallel porous disks one of which is rotating

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The steady magnetohydrodynamic laminar source flow of an incompressible conducting fluid between two parallel porous disks one of which is rotating and other is stationary in the presence of a transverse magnetic field has been analytically investigated. The solution has been obtained by expanding the velocity components and pressure in a power series of reduced Reynolds number $R_e/\bar{r}^2$, where R_e is the source Reynolds number and $\bar{r}$ is the radial distance. The effect of the magnetic field on the velocity profiles is to flatten them near the central region between the disks. The skin-friction, torque and pressure drop in the radial direction increase with the increase of Hartmann number M with the exception that the skin-friction decreases when $1 < M < 2$ and the torque decreases when $0 < M < 1$.

1. INTRODUCTION

The problem of flow between two parallel disks is of general interest because of its important in many engineering applications. Peube (1963) and Savage (1964) studied the source flow between stationary non-porous disks while Elkouh (1969, 1971) considered the same problem in the presence of suction and/or injection at the disks. Kreith & Viviand (1967) investigated the source flow between two non-porous disks which are rotating with different angular velocities. Khan (1968) also discussed the source flow between two porous disks rotating with the same speed with suction at one disk and equal injection at the other.

Srivastava & Sharma (1961) studied the magnetohydrodynamic (MHD) flow between a slowly rotating disk and a stationary disk. Stephenson (1969) has extended the above problem for rotating perfectly conducting disks. The MHD flow between two porous disks has been investigated by Chandrasekhar & Rudraiah (1970, 1971) firstly for stationary disks and subsequently for one rotating and other stationary disk. Recently, Khader & Goodling (1972) have studied the MHD laminar source flow between two parallel stationary disks.

In the present paper, we have discussed the MHD laminar source flow between two co-axial parallel porous disks one is rotating and other is stationary. The problem has been solved by applying the technique of double expansion of velocity components and pressure, firstly for a weak source midway between the disks and

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secondly under the assumption of small suction and slow rotation. It is observed that the effect of the magnetic field on the velocity profiles is to push them towards the central region between the disks so that they are flattened. The skin-friction on the disks increases with the increase of Hartman number M when $0 < M < 1$ and $M < 2$, and decreases when $1 < M < 2$. The torque on the disks increases with increasing $M > 1$ and decreases when $0 < M < 1$. The pressure drop in the radial direction increases with the increase of M.

2. Basic Equations

Consider a steady laminar flow of a viscous incompressible conducting fluid of density ρ, viscosity μ and electrical conductivity σ between two non-conducting porous disks separated by a distance $2a$ with a source of strength Q at the centre of the disks. The disk at $\bar{z} = a$ rotates with an angular velocity Ω and the disk at $\bar{z} = -a$ is stationary (figure 1). The fluid is injected with the uniform velocity V at both disks. A uniform magnetic field H_0 is applied in the $\bar{Z}$-direction.

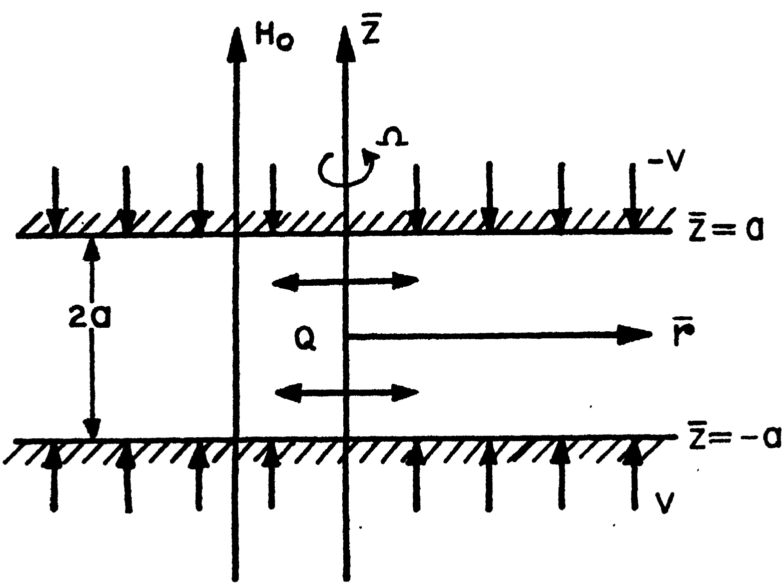

Figure 1. Physical model

The governing MHD equations of motion for steady flow, are

$$\rho(\boldsymbol{q}\cdot\nabla)\boldsymbol{q} = -\nabla p+\mu\nabla^2\boldsymbol{q}+\boldsymbol{J}\times\boldsymbol{B}, \quad \ldots (1)$$

$$\nabla\cdot\boldsymbol{q} = 0, \quad \ldots (2)$$

$$\nabla\times\boldsymbol{E} = 0, \quad \ldots (3)$$

$$\boldsymbol{J} = \sigma[\boldsymbol{E}+\boldsymbol{q}\times\boldsymbol{B}], \quad \ldots (4)$$

where $\boldsymbol{B} = \mu_e\boldsymbol{H}$, μ_e being magnetic permeability.

Neglecting the induced magnetic field, the components of velocity and magnetic field can be taken as $q(\bar{u}_r, \bar{u}_\theta, \bar{u}_z)$ and $B(0, 0, B_0)$ respectively.

For axisymmetric flow eq (3) gives $E_\theta = 0$ throughout the fluid. Following the analysis of Stephenson (1969), we get E_r, function of $\bar{r}$ and $E_z =$ function of $\bar{z}$. Assuming $E_r = -\bar{\chi}B_0\Omega\bar{r}$ and using the modified Ohm's law we get the components of current density as

$$\begin{aligned} J_r &= \sigma B_0(\bar{u}_\theta - \bar{\chi}\Omega\bar{r}), \\ J_\theta &= -\sigma B_0\bar{u}_r, \\ J_z &= \sigma E_z, \end{aligned} \tag{5}$$

where $\bar{\chi}$ is a dimensionless quantity representing the strength of the induced radial electric field and is equal to $\frac{1}{2a\Omega}\int_{-a}^{a}(\bar{u}_\theta/\bar{r})d\bar{z}$ for non-conducting disks.

Introducing the non-dimensional quantities defined by

$$r = \bar{r}/a, \quad z = \bar{z}/a, \quad u_r = \bar{u}_r a/\nu, \quad u_\theta = \bar{u}_\theta a/\nu,$$

$$u_z = \bar{u}_z a/\nu, \quad p = \frac{\bar{p}a^2}{\rho\nu^2}, \quad \chi = \frac{\bar{\chi}\Omega a^2}{\nu}$$

and using eq. (5), eqs. (1) and (2) can be written in cylindrical polar co-ordinates as

$$u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} = -\frac{\partial p}{\partial r} + D^2u_r - \frac{u_r}{r^2} - M^2u_r, \qquad \dots (7)$$

$$u_r\frac{\partial u_\theta}{\partial r} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_ru_\theta}{r} = D^2u_\theta - \frac{u_\theta}{r^2} - M^2(u_\theta - \chi r), \qquad \dots (8)$$

$$u_r\frac{\partial u_r}{\partial r} + u_r\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial z} + D^2u_z, \qquad \dots (9)$$

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} = 0, \qquad \dots (10)$$

where $$D^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}.$$

The boundary conditions can be written as

$$u_r(r, \pm 1) = 0, \quad u_\theta(r, 1) = \alpha r, \quad u_\theta(r, -1) = 0, \qquad \dots (11a)$$

$$u_z(r, \pm 1) = \mp R_w,$$

and $$\int_{-1}^{1} u_r dz - rR_w = \frac{2R_e}{r}, \qquad \dots (11b)$$

where $R_w = \frac{Va}{\nu}$ is the wall Reynolds number; $M^2 = B_0{}^2a^2 \frac{\sigma}{\rho\nu}$, the square of the Hartmann number; $\alpha = \frac{\Omega a^2}{\nu}$, the rotational Reynolds number; $R_e = Q/4\pi\nu a$, the source Reynolds number and eq. (11b) represents the overall conservation of mass.

In order to obtain the solution of the problem, we assume the following series expansions which are valid for small values of $R_e{}^*(= R_e/r^2)$:

$$u_r = \tfrac{1}{2}rf_1'(z)+\frac{R_e}{r}\,[f_2'(z)+R_e{}^*f_3'(z)+R_e{}^{*2}f_4'(z)+\ldots] \qquad \ldots \quad (12)$$

$$u_e = rg_1(z)+\ \frac{R_e}{r}\,[g_2(z)+R_e{}^*g_3(z)+R_e{}^{*2}g_4(z)+\ldots] \qquad \ldots \quad (13)$$

$$u_z = -f_1(z)+2R_e{}^{*2}f_3(z)+4R_e{}^{*3}f_4(z)+\ldots \qquad \ldots \quad (14)$$

$$\chi = \chi_1+R_e{}^*\chi_2+R_e{}^{*2}\chi_3+R_e{}^{*3}\chi_4+\ldots, \qquad \ldots \quad (15)$$

and

$$p = \{\tfrac{1}{4}r^2h_1(z)+K(z)\}+R_e[h_2(z)\log r+R_e{}^*h_3(z)+\ldots]. \qquad \ldots \quad (16)$$

Substituting eqs. (12) -(16) for u_r, u_θ, u_z, χ and p in eqs. (7)-(9) and equating the coefficients of like powers in r, the Navier-Stokes equations are reduced to to an infinite set of systems of simultaneous ordinary differential equations. The first two systems are

System 1

$$\begin{aligned} f_1'''+f_1f_1''-\tfrac{1}{2}f_1'^2+2g_1{}^2 &= h_1+M^2f_1', \\ g_1''+f_1g_1'-f_1'g_1 &= M^2(g_1-\chi_1), \\ h_1' &= 0, \\ K' &= -f_1''-f_1f_1'. \end{aligned} \qquad \ldots \quad (17)$$

System 2

$$\begin{aligned} f_2'''+f_1f_2''+2g_1g_2 &= h_2+M^2f_2', \\ g_2''+f_1g_2'-2gf_2' &= M^2(g_2-\chi_2), \\ h_2' &= 0. \end{aligned} \qquad \ldots \quad (18)$$

The corresponding boundary condition can be written as

$$\left.\begin{aligned} &f_1'(\pm1) = f_2'(\pm1) = 0, \quad f_1(\pm1) = \pm R_w, \\ &g_1(+1) = \alpha,\ g_1(-1) = 0, \quad g_2(\pm1) = 0. \end{aligned}\right\} \qquad \ldots \quad (19a)$$

and

$$\left.\begin{aligned} &f_2(+1)-f_2(-1) = 2, \text{ but choosing} \\ &f_2(+1) = 1 \quad \text{gives} \quad f_2(-1) = -1. \end{aligned}\right\} \qquad \ldots \quad (19b)$$

3. Solution of the Equations

(a) *System* 1

In order to obtain a solution to system 1, we assume a series solution in powers of α and R_w which is valid for small values of rotational Reynolds number α and wall Reynolds number R_w, in the form

$$
\begin{aligned}
f_1 &= \alpha f_{10}+R_w f_{11}+\alpha^2 f_{12}+R_w^2 f_{13}+2\alpha R_w f_{14}+\dots,\\
g_1 &= \alpha g_{10}+R_w g_{11}+\alpha^2 g_{12}+R_w^2 g_{13}+2\alpha R_w g_{14}+\dots,\\
h_1 &= \alpha h_{10}+R_w h_{11}+\alpha^2 h_{12}+R_w^2 h_{13}+2\alpha R_w h_{14}+\dots,\\
\chi_1 &= \alpha \chi_{10}+R_w \chi_{11}+\alpha^2 \chi_{12}+R_w^2 \chi_{13}+2\alpha R_w \chi_{14}+\dots .
\end{aligned} \qquad \dots (20)
$$

Using these in eq (17) and equating the like powers of α and R_w upto the second order. We get the first five sets of ordinary linear differential equations whose solutions under the appropriate boundary conditions are

$$f_1 = R_w\left[B_1 \sinh MZ-\frac{h_{11}}{M^2}Z\right]+\alpha^2\left[B_2\cosh MZ+B_3\sinh MZ+B_4-\right.$$

$$\left.-B_5 Z-\frac{A_1^2}{6M^3}\sinh 2MZ-\frac{A_1}{M^2}Z\sinh MZ\right]+R_w^2\left[B_6\sinh MZ-\right.$$

$$\left.-B_7 Z-\frac{B_1h_{11}}{4M^3}(5Z\cosh MZ-MZ^2\sinh MZ)-\frac{B_1^2}{24M}\sinh 2MZ\right],$$

$$g_1 = \alpha[A_1\sinh MZ+1/2]+2\alpha R_w[A_2\cosh MZ+A_3\sinh MZ+\frac{B_1}{8}Z\sinh MZ$$

$$-\frac{A_1h_{11}}{8M^3}(3Z\cosh MZ-MZ^2\sinh MZ)+\frac{h_{11}}{4M^4}+\chi_{14}\Big],$$

$$h_{10} = h_{14} = 0,\quad h_{11} = B_1M^3\cosh M,$$

$$h_{12} = B_3M^3\cosh M-\frac{A_1^2}{3}(3+\cosh 2M)+\tfrac{1}{2},$$

$$h_{13} = B_8\cosh M-\frac{B_1^2M^2}{12}\cosh 2M-B_9-\tfrac{3}{4}B_1h_{11}\sinh M,$$

$$\chi_{10} = 1/2,\quad \chi_{11} = \chi_{12} = \chi_{13} = 0,$$

$$\chi_{14} = -A_2\cosh M-\frac{B_1}{8}\sinh M-\frac{h_{11}}{4M^4},$$

where

$$A_1 = \frac{1}{2\sinh M}, \quad B_1 = (\sinh M - M\cosh M)^{-1}, \quad A_2 = \frac{M^2-2h_{11}}{8M^3\sinh M},$$

$$B_2 = \frac{A_1}{M^3\sinh M}(\sinh M + M\cosh M), \quad B_3 = \frac{B_1 A_1^2}{6M^3}(\sinh 2M - 2M\cosh 2M),$$

$$B_4 = \frac{A_1}{M^2}\sinh M - B_2\cosh M, \quad B_5 = \frac{1}{M^2}(h_{12}+A_1^2-1/2)$$

$$A_3 = \frac{A_1 h_{11}}{8M^3\sinh M}(3\cosh M - M\sinh M),$$

$$B_6 = \frac{B_1^2}{24M}(\sinh 2M - 2M\cosh 2M) - \frac{B_1^2 h_{11}}{4M^2}(4\sinh M - M\cosh M),$$

$$B_7 = \frac{h_{13}}{M^2} + \frac{3B_1^2}{4} + \frac{h_{11}^2}{M}, \qquad B_8 = M^3 B_6 + \frac{B_1 h_{11}(M^2-5)}{4M},$$

$$B_9 = \frac{M^2}{2}\left(\frac{3B_1^2}{2} + \frac{h_{11}^2}{M^6}\right).$$

The solution of system 1 represents the source free flow between two parallel porous disks one is rotating and other is stationary. Substituting $\alpha = 0$ in eq. (20) one gets the solution for the two dimensional flow between two parallel stationary porous disks investigated by Chandrasekhara & Rudraiah (1970).

(*b*) *System* 2

Similarly, to obtain the solution of system 2, we assume the series solutions in powers of α and R_w as

$$\begin{aligned} f_2 &= f_{20}+\alpha f_{21}+R_w f_{22}+\alpha^2 f_{23}+R_w^2 f_{24}+2\alpha R_w f_{25}+\dots, \\ g_2 &= g_{20}+\alpha g_{21}+R_w g_{22}+\alpha^2 g_{23}+R_w^2 g_{24}+2\alpha R_w g_{25}+\dots, \qquad \dots \quad (21) \\ h_2 &= h_{20}+\alpha h_{21}+R_w h_{22}+\alpha^2 h_{23}+R_w^2 h_{24}+2\alpha R_w g_{25}+\dots, \\ \chi_2 &= \chi_{20}+\alpha\chi_{21}+R_w\chi_{22}+\alpha^2\chi_{23}+R^2{}_w\chi_{24}+2\alpha R_w\chi_{25}+\dots. \end{aligned}$$

Substituting eq. (21) for f_2, g_2, h_2 and χ_2 in eq. (18) and collecting the coefficient of α and R_w upto the 2nd order terms, we get six sets of ordinary differential equations whose solutions under the appropriate boundary conditions are

$$f_2 = \left(B_1\sinh MZ - \frac{h_{20}}{M^2}Z\right) + R_w\Big[C_1\sinh MZ - \frac{h_{22}}{M^2}Z - \frac{B_1^2}{12M}(6MZ+$$

$$+\sinh 2MZ) + \frac{B_1 h_{11}}{4M^3}(MZ^2\sinh MZ - 3Z\cosh MZ)\Big]+$$

$$+\alpha^2\Big[C_{11}\cosh MZ+C_{12}\sinh MZ+C_{13}-\Big(C_8+\frac{h_{23}}{M^2}\Big)Z-$$

$$-\frac{C_2}{6M^3}\cosh 2MZ-C_7\sinh 2MZ-C_6Z\sinh MZ+$$

$$+\frac{C_5}{4M^3}Z^2\sinh MZ-\frac{A_1{}^2B_1}{96M^4}\sinh 3MZ-C_9Z\cosh MZ+C_{10}Z^2\cosh MZ+$$

$$+\frac{A_1{}^2h_{20}}{6M^6}\,Z\cosh 2MZ\Big]+R_w{}^2\Big[-C_{16}\sinh 2MZ-\Big(C_{17}+\frac{h_{24}}{M^2}\Big)Z+$$

$$+C_{18}\sinh 3MZ-C_{19}Z\cosh MZ-C_{20}Z\cosh 2MZ+C_{21}Z^2\sinh MZ-$$

$$-\frac{C_{14}}{12M^3}\,(Z^2\sinh 2MZ+2MZ^3)+C_{22}Z^3\cosh MZ+C_{23}Z^4\sinh MZ\Big],$$

$$g_2=\alpha\Big[A_4\cosh MZ+A_5\sinh MZ+\frac{A_1B_1}{3M}\sinh 2MZ-\frac{A_1h_{20}}{M^3}\,Z\cosh MZ+$$

$$+\frac{B_1}{2}(Z\sinh MZ-\sinh M)-A_4\cosh M\Big]+2\alpha R_w\Big[T_8\cosh MZ+$$

$$+T_9\sinh MZ+A_{10}\cosh 2MZ+A_{11}+A_{12}\sinh 2MZ-A_{13}Z\cosh MZ+$$

$$+A_{14}Z\cosh 2MZ-\frac{A_6}{16M^2}\sinh 3MZ+A_{15}Z+\frac{A_7}{3M^2}Z^2\sinh 2MZ+$$

$$+A_{16}Z\sinh MZ+A_{17}Z^2\cosh MZ-A_{18}Z^2\sinh MZ+$$

$$+\frac{A_8}{6M}\,Z^3\sinh MZ-\frac{A_9}{6M}\,Z^3\cosh MZ+\chi_{25}\Big],$$

$$h_{20}=B_1M^3\cosh M,\quad h_{21}=h_{25}=0,$$

$$h_{22}=C_1M^3\cosh M-\frac{B_1{}^2M^2}{6}(\cosh 2M+3)+\frac{B_1h_{11}}{4}(M\cosh M+2\sinh M)-$$

$$-\frac{3B_1h_{11}}{4M}(\cosh M+M\sinh M),$$

$$h_{23}=T_1\cosh M-T_2\cosh 2M-M^2C_8+T_3\sinh M-\frac{B_1A_1{}^2}{32M}\cosh 3M+$$

$$+\frac{A_1{}^2h_{20}}{3M^3}\sinh 2M,$$

$$h_{24}=T_4\cosh M-T_5\cosh 2M-\Big(M^2C_{17}+\frac{C_{14}}{2}\Big)+3M^2C_{18}\cosh 3M+$$

$$+T_6\sinh M-T_7\sinh 2M.$$

$$\chi_{20} = \chi_{22} = \chi_{23} = \chi_{24} = 0,$$

$$\chi_{21} = -A_4 \cosh M - \frac{B_1}{2} \sinh M - \frac{h_{20}}{M^4},$$

$$\chi_{25} = -\left[(T_8 + A_{17}) \cosh M + A_{10} \cosh 2M + A_{11} + \left(A_{16} + \frac{A_8}{6M}\right) \sinh M\right],$$

where the constants $C_1, C_2, C_5, C_6, C_7, \ldots, C_{23}, A_4, A_5, A_6, \ldots, A_{18}, T_1, T_2, \ldots, T_9$ are dependent on magnetic parameter M.

The solution of system 2 represents the source flow. The first term of solution (21) corresponds to inertialess flow without any rotation and suction investigated by Khader & Goodling (1972). If we take $\alpha = 0$,, the solution represents the source flow between the porous disks without rotation.

4. Discussion of Results

The radial and azimuthal components of velocity are given by

$$\frac{u_r}{r} = \tfrac{1}{2} f_1'(z) + R_e^* f_2'(z) + O(R_e^{*2}),$$

and

$$\frac{u_\theta}{r} = g_1(z) + R_e^* g_2(z) + O(R_e^{*2}). \qquad \ldots (22)$$

The radial and azimuthal velocity profiles depend on the parameters α, R_w and R_e^*. The profiles of the radial and azimuthal components of velocity have been plotted in figures 2 and 3 for various values of M, α and R_w and for a value of $R_e^* = 10.00$. From figure 2, it is observed that for a given value of M, increase of α and R_w increases the magnitude of radial velocity and in figure 3, it is seen for a given value of M the magnitude of azimuthal velocity increases with the increase of R_w.

From these figures, we can also conclude that the magnitude of the radial and azimuthal components of velocity decreases with the increase of magnetic field and the effect of the magnetic field on the velocity profiles is to flatten them near the central region of the disks.

For practical purposes, it is important to find the skin-friction and torque on the disks. The skin-griction on the disks is obtainable from the shearing stress component τ_{zr} which is given by

$$\tau_{zr} = \mu \frac{\partial \bar{u}_r}{\partial \bar{z}} = \frac{\mu \nu}{a^2} \frac{\partial u_r}{\partial z} \qquad \ldots (23)$$

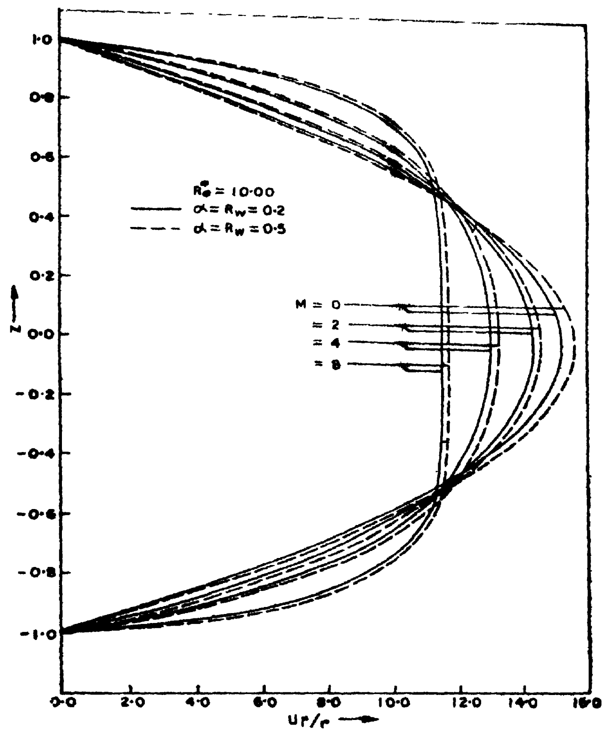

Fig. 2. Non-dimensional radial velocity distribution ($u_{r/r}$).

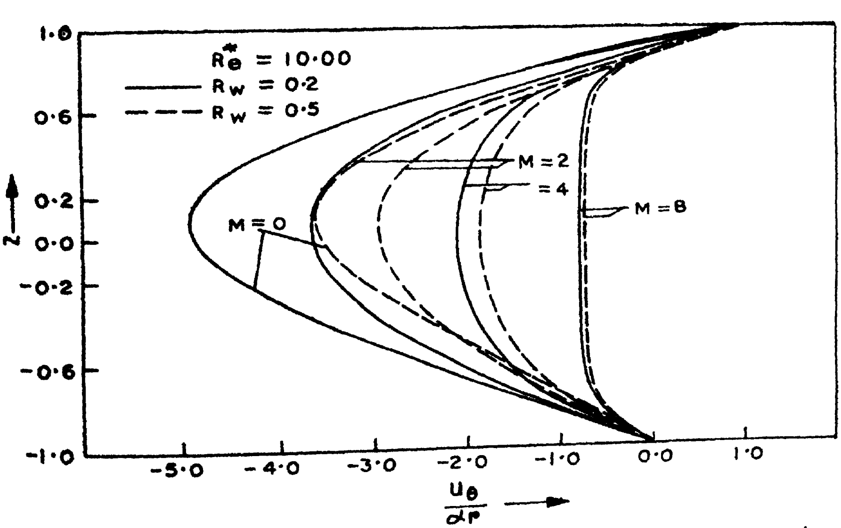

Fig. 3. Non-dimensional azimuthal velocity distribution ($u_\theta/\alpha r$).

The dimensionless skin-friction on the disks ($z = \pm 1$) is

$$\tau^*|_{z=\pm 1} = -\left[\tfrac{1}{2} r f_1''(\pm 1) + \frac{R_e}{r} f_2''(\pm 1)\right], \quad \dots (24)$$

where $$\tau^* = \bar{\tau}_{zr}\left(\frac{a^2}{\mu\nu}\right).$$

The dimensionless skin-friction (τ^*/r) on the upper disk ($z = 1$) has been plotted against the magnetic parameter M in figure 4. From this, it is observed that the skin-friction increases with increasing M when $M > 2$. The behaviour of skin-friction for $M < 2$ is that, firstly it increases when $0 < M < 1$ and then decreases when $1 < M < 2$.

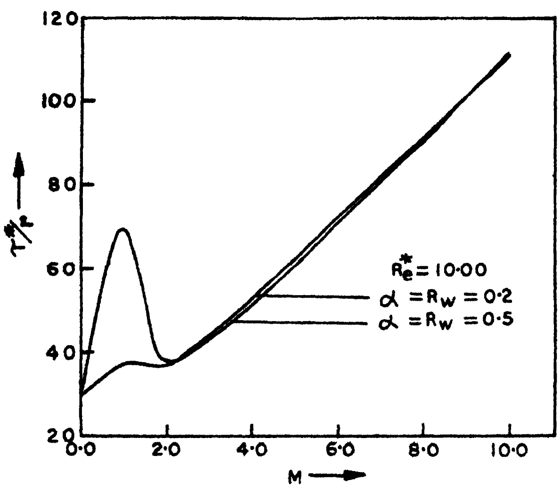

Fig. 4. Non-dimensional skin-friction against magnetic parameter M.

The torque on the disks depends on the shearing stress component $\bar{\tau}_{z\theta}$ which acts in the plane of the disk and is given by

$$\bar{\tau}_{z\theta} = \mu \frac{\partial \bar{u}_\theta}{\partial \bar{z}} = \frac{\mu\nu}{a^2}[r g_1' + (R_e/r) g_2']. \quad \dots (25)$$

Neglecting the edge effects the torque on a finite disk of radius $\bar{R}$ is

$$\bar{\tau} = \int_0^{\bar{R}} 2\pi \bar{r}^2 \bar{\tau}_{z\theta} d\bar{r} = 2\pi\mu\nu a \int_0^{R} r^3[g_1' + (R_e/r^2) g_2'] dr, \quad \dots (26)$$

where $R = \bar{R}/a$.

Hence the dimensionless torques on the disks ($z = \pm 1$) are

$$\tau|_{z=\pm 1} = g_1'(\pm 1)+2\,\frac{R_e}{R^2}\,g_2'(\pm 1), \qquad \dots \quad (27)$$

where $$\tau = \frac{2\bar{\tau}}{R^4 \pi \mu \nu a}\,.$$

From eq. (27), we can find the torques on both the disks but the torque on the upper disk only has been calculated for $(R_e/R^2) = 5.0$ and presented in figure 5. From the figure, it is found that the torque on the upper disk increases with the increase of magnetic parameter M when $M > 1$, but it decreases when $0 < M < 1$.

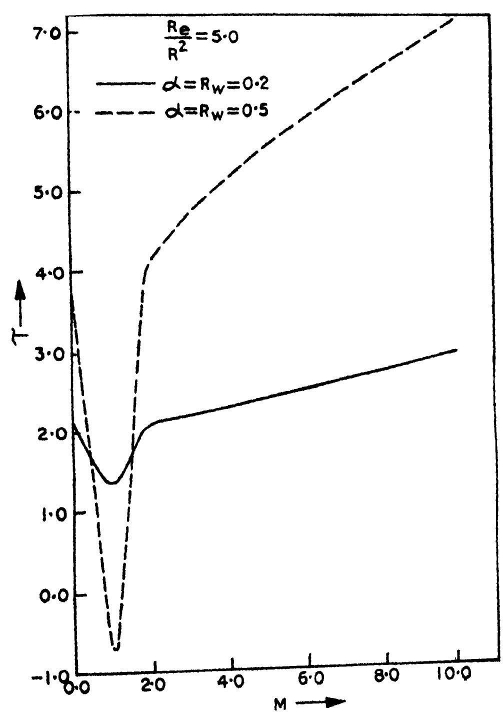

Fig. 5. Graph of non-dimensional torque τ against magnetic parameter M.

The pressure drop in the radial direction at any point ($r = R$) in the flow given by

$$\begin{aligned} p^* &= p(r)-p(R) \\ &= 1/4(r^2-R^2)h_1+R_e \log (r/R)h_2. \end{aligned} \qquad \dots \quad (28)$$

The pressure drop in the radial direction has been plotted against r in figure 6 for various values of parameters α, R_w and M. From this, it is observed that pressure drop p^* increases with the increase of M.

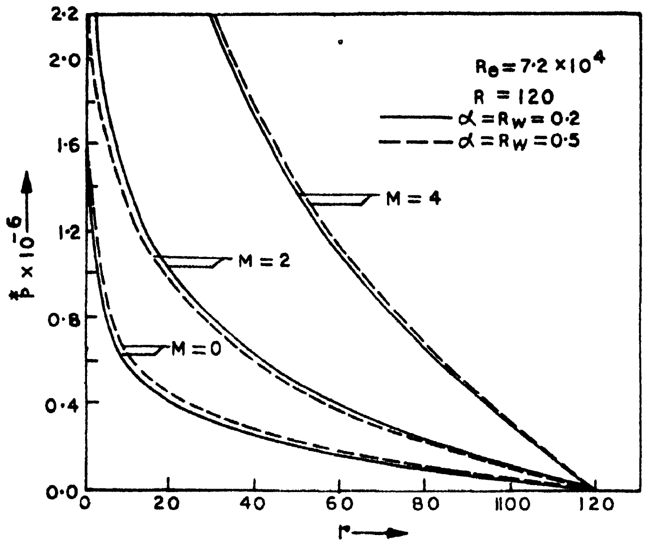

Fig. 6. Non-dimensional pressure drop p^* in the radial direction.

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